Distributed Heuristics for Allocating Spectrum in CR Ad hoc Networks

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Abstract—Cognitive Radios (CRs) address spectrum scarcity and under-utilization of the spectrum. However, realizing a CR network is neither easy nor straight-forward. In particular the link layer should facilitate setting up of communication between nodes enabling sharing of channels, while maximizing the spectrum utilization. We address the problem of allocation of channels to CR nodes. For time-slotted model, allocating the channels among CR nodes is an \( NP \) complete problem. We propose novel distributed heuristics, one based on clique sets called Clique Based Heuristic and the second one is a low complexity, locally adapting Localized Heuristic; both use local control channel. We compare the results of these heuristics with the optimal and a distributed graph coloring algorithm proposed in the literature. We also compare the spectrum utilization and overheads for all these algorithms.

I. INTRODUCTION

It is predicted by [1] that there would be 1000 wireless devices per person on an average. Thus, the density of wireless devices will increase around a person and mainly inside homes. Many appliances and supporting devices, sensors and actuators, will operate and coordinate wirelessly. Thus these in-home networks in turn will increase the need of spectrum. However, the current regulatory practice of licensing frequency bands exclusively for long periods of time has resulted in overcrowding of some bands while many licensed bands are under-utilized. This has created an artificial scarcity of the spectrum. Spectrum utilization varies between 15% to 85% as reported by FCC [2]. DARPA reports an average of only 6% licensed spectrum usage [3]. Cognitive Radios (CRs) are expected to address this scarcity of spectrum. CRs can detect unused spectrum bands (spectrum holes), and can access these holes opportunistically. With Opportunistic Spectrum Access (OSA) DARPA [3] envisages to increase the spectrum utilization 10 folds. A Cognitive Radio Ad hoc Network (CRAN) is an ad hoc network formed by CR equipped nodes. Fig. 1 shows components of a CRAN architecture [4]. Primary network provides services to Primary Users (PU) over a certain licensed spectrum with Primary base-stations. CRAN nodes, called Secondary Users (SUs), can opportunistically access spectrum bands (both licensed and free bands) for communication. Few envisaged applications of CRANs are: (a) textual and multimedia services and access to IP services in a vehicular environment [5]; (b) military communications; (c) public safety and emergency services [6] [7]; and (d) CR mesh networks for broadband home networking, community networking, etc. To realize CRANs and its applications, a coordinated access of spectrum is required by SUs. Spectrum sharing in CRANs is a challenging task due to many dynamically changing available set of channels and number of SUs that vary over time and space, with no central entity to coordinate. Moreover, the SUs should abide by spectrum usage policies and etiquettes in force. Following are the two most important challenges to achieve a coordinated sharing of spectrum [4] [8].

- **Spectrum coordination and access:** Every (transmitter, receiver) pair should be aware of which channel to use and time of its access. This handshake is crucial and needs to take place before the start of data transmission.
- **Spectrum changes:** The CR nodes should adapt to the changes in availability of channels, due to PU activity, causing least amount of interference to PUs.

A spectrum sharing framework for CRANs should address the above two challenges without a central controller and should constantly strive to increase spectrum utilization. We propose distributed, allocation in slotted time frames. Allocation and distribution of the schedule ensures the spectrum coordination and access. We explore novel graph theoretic approaches to achieve these objectives. The article is organized as follows. In Sec. II, we summarize the closest work to this one in literature. In Sec. III we describe the problem of spectrum allocation and access in time-slotted system. In Sec. IV we propose heuristics for spectrum sharing. In Sec. V, we compare the performance of proposed heuristics with the optimal solution and a heuristic proposed in [9]. We conclude in Sec. VI.

II. RELATED WORK

While we propose a novel clique based solution, graph coloring for channel allocation has been investigated exten-
sively in literature for cellular networks, wireless networks and CDMA networks [10], [11]. Graph coloring is an $NP$ complete problem [12]. Several edge coloring heuristics have been proposed for link scheduling in multi-hop wireless networks [13], [11]. These solutions if applied to CRANs results in degraded performance, since the channel availability at each node varies. Here we summarize works [13], [11]. These solutions if applied to CRANs results have been proposed for link scheduling in multi-hop wireless networks [13], [11]. Graph coloring is an $NP$ complete problem [12]. Several edge coloring heuristics have been proposed for assignment of a unique < timeslot, channel > pair to the links (edges). An assignment with no hidden terminals is sought. The ILP based approach has high computational complexity and does not scale well, but produces an optimal assignment. The proposed distributed heuristic is based on edge coloring. Every node is assumed to be aware of its two hop neighbors. A global control channel is also assumed. The heuristic involves two phases. In Phase-1, a < timeslot, channel > pair is assigned to every link, and in Phase-2, the schedule length is propagated through the network. In Phase-1, nodes are ranked based on its degree or number of available channels, or a combination. A node with the highest rank in its two hop neighborhood and having edges not colored yet, starts the assignment process. After every edge is colored the node is said to be covered. This assignment is distributed amongst its two hop neighbors and the procedure continues till all the nodes are covered. This ensures a conflict-free allocation. In this distributed greedy heuristic, the procedure is executed sequentially, thus requires more time. In the process of conflict-free assignment, the schedule length increases. While it is necessary to get a conflict-free schedule, in our work we do not avoid the conflicts but we resolve them. This results in overall lower schedule length, and increases the spectrum usage.

III. PROBLEM DESCRIPTION

The spectrum is a shared resource amongst the nodes that are in each other’s interfering range. Unlike the MAC protocols of ad hoc networks, the MAC for CRANs should handle multiple channels at the same time. Unlike the multi-channel ad hoc networks, the CRANs should be able to adapt to the dynamic PU activities. The MAC for CRANs can be used in multi-channel networks, however applying multi-channel for CRANs results in degraded performance. In this work, we assume that the SU nodes have only one radio front-end. Each SU node having data to communicate should get access to at least one channel for transmission. Two important conditions we start with are: (i) Allocation: every (transmitter, receiver) pair must get access to a channel; and (ii) Access: the receiver must know the channel and time at which the transmitter will transmit. We choose and propose a TDMA based allocation algorithms for CRANs since it can completely satisfy the above requirements. Another advantage of such slotted schemes is the ability to address the issue of spectrum changes due to a coordinated spectrum sensing. We model the CRAN as an undirected graph.

A. Problem formulation

Let a CRAN be represented by an undirected graph $G = (V, E)$, with no self-loops, where $V$ and $E$ represent the set of vertices/nodes and links. A link exists between two vertices $x$ and $y$ if both $x$ and $y$ are in communicating radii of each other and at least one common channel is available to them. The spectrum allocation problem is to find an optimal valid assignment of < timeslot, channel > pair for every link.

$$A(x,y), j,k = \begin{cases} 1 & \text{if edge } (x,y) \text{ uses channel } j \text{ in time-slot } k \\ 0 & \text{otherwise} \end{cases}$$

The following are the necessary and sufficient conditions for a valid assignment.

1. $x$ and $y$ are in communication radii of each other,
2. both $x$ and $y$ are not involved in any other communication in $TS_k$,
3. assignment of channel $j$ does not cause interference to neighboring nodes.

Let $A$ represent an adjacency matrix of the allocation such that,

$$A(x,y) = \begin{cases} (k,j) & \text{if } A(x,y), j,k = 1 \\ 0 & \text{otherwise} \end{cases}$$

Let $A$ denote the set of all possible valid assignments for $A$. The length of an allocation $Len(A)$, is equal to the number of slots in the allocation. The allocation $A_{opt}$ is optimal if the $Len(A_{opt}) = min\{Len(A_i) | \forall A_i \in \Lambda \}$. The important symbols used are summarized in Table I.

It is necessary to find a minimum length MAC schedule for better channel reuse and higher network throughput [14]. The problem of spectrum allocation is $NP$ complete (see [15] for proof), and hence we propose heuristics. An Integer Linear Program (ILP) formulation of the problem is also presented in [9] to find an optimal solution.

<table>
<thead>
<tr>
<th>TABLE I DEFINITIONS</th>
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<tbody>
<tr>
<td>$G$</td>
</tr>
<tr>
<td>$N$</td>
</tr>
<tr>
<td>$(x, y)$</td>
</tr>
<tr>
<td>$TS_k$</td>
</tr>
<tr>
<td>$M$</td>
</tr>
<tr>
<td>$ChSet(x, y)$</td>
</tr>
<tr>
<td>$NodeID_x$</td>
</tr>
<tr>
<td>$\delta_x$</td>
</tr>
<tr>
<td>$Neighbors_x$</td>
</tr>
</tbody>
</table>

B. System model and assumptions

We consider a multi-hop CR network formed by $N$ nodes with unique node-ids $\{1, 2, ..., N\}$. These nodes have a half-duplex software-defined radio transceiver. These nodes are interested in only 1-hop communication. We assume that the spectrum is divided into $M$ orthogonal symmetric channels having a unique number in $\{1, 2, ..., M\}$. Further, we assume availability of a common control channel (CCC) to exchange control messages. The nodes are time synchronized through the CCC. We assume that all nodes have data to transmit to...
all its neighbors to ensure that we consider the worst case scenario of the problem. Consider the MAC super frame as shown in Fig. 2. The super frame is divided into sensing, information exchange, allocation and data transmission periods. In the sensing period, the channels that are free of PUs are found. We assume perfect sensing by nodes for the sake of simplicity. The nodes then send the list of channels available to their neighbors during the information exchange period. In the spectrum allocation period, an algorithm is executed to determine the schedule and the schedule is distributed. The data transmission period is split into $K$ equal length time-slots. The value of $K$ is determined from the topology of the CRAN under consideration using a bound. Due to lack of space, we omit the discussion here. In a time-slot $T_{S_k}$ both the participating nodes transmit and receive data i.e., the timeslot is considered to be consisting of two sub-slots. In the first sub-slot, one node transmits and the second receives, and in the second sub-slot, the receiver in the first sub slot transmits its data to its counterpart. The sub-slots may be of unequal lengths depending on the data that each node has to transmit to the other. This assumption reduces the complexity in the design, since each link has to be assigned with one time-slot. We also note here that we have assumed in the beginning that all the nodes have data to be transmitted to all other nodes to get the worst case.

We neglect channel switch times since we only propose solution for allocation but it could be easily captured by reserving a constant time slice in the data transmission period. For time synchronization, a scheme such as [16] can be used. In the current state of the art, nodes cannot independently and accurately determine the spectrum holes. However for simplicity, we assume (a) accurate sensing (with or without cooperative sensing) and (b) PUs change states only at the beginning of the frame.

IV. PROPOSED HEURISTICS

A. Clique Based Heuristic (CBH)

A clique in an undirected graph $G = (V, E)$ is a subset of the vertices, $V' \subseteq V$, such that for every two vertices in $V'$ there exists an edge connecting the two. The complement of a clique is an independent set, wherein every clique corresponds to an independent set. In CBH, two hops topology information is exchanged between the nodes. Since we are attempting a distributed algorithm, we should take care that no edge is assigned with more than one (timeslot, channel) pair that creates conflicts. Hence each edge is handled by one node to fix the responsibilities, each node executes the Procedure 1. Let $I$, denote an Independent Set of $G$. At the end of the procedure, we have a set of nodes that form a maximal independent set $(I)$ of the graph. Each node can independently determine which node in its 1-hop neighborhood is in $I$ with two hop topology information. The nodes not in $I$, i.e., $\bar{I}$, assign (timeslot, channel) to those edges for which the other end is in $I$ or if other node has higher node-id. The assignment is described in Procedure 2. With maximal independent set, the number of conflicts is less. Once the assignment is done, each node exchanges the schedule in its one hop neighborhood. Conflicts may arise since each node, in $I$, executes this procedure independently. If two edges have conflicting schedule, it is not required by all 4 end-vertices to find a new allocation; this only complicates the issue at hand. In some cases, only 2 nodes may detect the conflict since we check for the conflicts in 1-hop neighborhood. Node with the least node-id amongst the conflicting vertices is responsible for carrying out conflict resolution. This node checks its allocation table, and first tries if it can solve the conflict by changing the channel assigned to the conflicting edge. If it is not possible, it finds an available slot or creates a new slot if required. This proposal for resolving conflict is sent to the other end-node of the conflicting edge. The other end-vertex also finds a new allocation; this only complicates the issue at hand. The decision is informed to all its 1-hop neighbors. The conflict resolving node also notes the decision if it results in a valid allocation, and broadcasts this information to 1-hop neighbors. This continues until all the edges have resolved the conflicts and a valid schedule are determined.

![Fig. 2. MAC super frame](image-url)
**Algorithm 4 Clique Based Heuristic**

1. **Input:** the set of 1-hop neighbors for each node $x$ and the set of available channels between each pair $x, y$ such that $y$ is a neighbor of $x$
2. **Output:** $A_{(x,y), j,k}$
3. $I \leftarrow \emptyset$
4. Run Procedure 1
5. Run Procedure 2
6. Distribute $A_x$ over 1-hop neighborhood
7. Run Procedure 3

is $O(M N \text{maxSlot})$, and for Proc. 3 is $O(N^2 M \text{maxSlot})$ at each node, where $\text{maxSlot}$ is the maximum number of slots used.

**B. Localized Heuristics**

In the previous algorithm, allocation procedure is executed at the beginning of every frame. To find a valid allocation, many control messages are exchanged even if there is no change in the scenario. In fact PU channels which are most likely to be inactive for long periods can be selected for allocation. Some SU nodes may see a topology change due to PU occupying all available channels between them, or SU nodes see a different set of channels available while the allocated channel gets occupied. Thus to reduce unnecessary execution and message exchanges, a new algorithm is proposed wherein, only the nodes that find changes in the scenario participate in the new allocation procedure. In these cases we want only those nodes that see the change to adapt to the environment. Thus we avoid performing a complete allocation at all nodes. The Localized Heuristic is given in Alg. 5. Note that input $A_{(x,y), j,k}^{\text{old}}$ from CBH, ILP or any other `parent` algorithm. The number of messages exchanged in this algorithm is less than that of CBH algorithm - firstly, there is no requirement to distribute the allocation to its neighbors (Step 6 of Alg. 4); secondly, the procedure is similar to Conflict Resolution procedure of CBH and messages are exchanged only if a node is affected by PU. Change in topology takes higher priority than channel change. In case a node experiences both, only topology change is taken into account since it will anyway reallocate channels to the edges. The complexity of this algorithm is $O(N^2 M \text{maxSlot})$, where $\text{maxSlot}$ is the maximum number of slots used, at every node. The detailed explanation of the algorithms can be found in [15].

**V. RESULTS AND DISCUSSIONS**

**A. Simulation environment and Experiments setup**

We performed simulations to test the algorithms for various metrics. The simulator creates a 2D area with four PUs with their coverage area as shown in Fig. 3(a). The dimensions of the area, maximum number of channels, and number of SUs are input by the user. The SUs are uniformly distributed over the area. Each PU has a call arrival rate of 36 calls/hour with a mean call duration of 80s. Due to the exponential off distribution of channel occupancy dynamic topologies of the SU nodes are created, the number of channels also varies between neighbors due to the path loss and shadowing; we have used lognormal model. Four simulation scenarios are considered as shown in Table II. In the first two scenarios, the number of nodes is chosen so as to maintain the same density for different areas. The number of channels are varied across
scenarios. In the next two scenarios, the area is kept constant to create sparse and dense graphs; again, the number of channels is varied across scenarios. We implemented a centralized edge coloring heuristic (referred to as CTA) [15] to compare performances with the distributed Clique Based Heuristic (CBH), and Localized Heuristics (LH). We considered two LH can be formed - one that takes input from CTA (LH CTA) and the other that takes input from CBH (LH CBH). The results are compared with MAC Scheduling proposed in [9]. LH CTA is partially decentralized i.e., the first allocation is centralized; CBH and LH CBH are completely distributed heuristics. All algorithms are run for 20 iterations with 100 allocations for all the scenarios. Due to lack of space, we show results for Scenario 4 which has the highest density of nodes and least number of channels.

### B. Schedule Length

The ILP formulation shown in [9] is solved using CPLEX [17] to compare different algorithms. Since CPLEX takes huge amount of time for a 90 node graph, a new experiment consisting of 25 nodes in an area of 158x158m² is added (the density of nodes is approximately the same)). For 90 nodes, we show the performance of algorithms without comparing them to ILP. Fig. 3(b) shows the mean schedule lengths of various heuristics and the optimal result from ILP in Scenario 4. For 10 nodes, there is no significant difference in the lengths. When 25, 40 and 90 nodes are considered, CTA outperforms the other heuristics since it has complete knowledge of the topology. CBH outputs slightly lower lengths than MAC Scheduling algorithm. The LH based algorithms produce lengths less than or equal to their parent algorithms. This is because, LH based schemes try to adapt to the new situation by making minimal changes. However when new assignments are sought, the greediness of the algorithms affects the lengths. In all the cases, the proposed heuristics are very close to the optimal result obtained from solving ILP equations, which indicates good performance of the heuristics.

### C. Utilization

Increasing spectrum utilization is one of the main goals in CRANs. Here we consider utilization of slots by nodes within one schedule length. A slot is utilized by a node if the node is allocated a channel in the slot. In the schedule it is possible that an edge not participating in the current timeslot, can be allocated with one of the available channels without affecting the validity of the schedule. Such a slot is counted as free-slot for both the end-vertices of the edge. This increases the spectrum utilization. The free-slots usage can either be predetermined and distributed with the schedule, or they may be found by the nodes listening on control channel, and then setting up the communication with other free neighbors. Here, in simulations, we run an algorithm for a possible free-slot allocation and the averages are presented here. The percentage

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**TABLE II**

<table>
<thead>
<tr>
<th>Scenario 1</th>
<th>Scenario 2</th>
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<tr>
<td>Area</td>
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</tr>
<tr>
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<td>40</td>
</tr>
<tr>
<td>300x300</td>
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**Scenario 3 | Scenario 4**

<table>
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<th>Area</th>
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<th>Channels</th>
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</tr>
<tr>
<td>200x200</td>
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<td>1-2</td>
</tr>
<tr>
<td>200x200</td>
<td>90</td>
<td>1-2</td>
</tr>
</tbody>
</table>

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Fig. 3. Simulation scenario and comparison of algorithms
of utilization per edge for Scenario 4 is shown in Fig. 3(c). MAC Scheduling proposed in [9] does not use the free-slots. The schedule, however, ensures that every edge is utilized once, hence the average utilization per edge is considered 1 for all the cases. The average utilization when using CBH is slightly higher since the schedule length produced by it is higher than the other algorithms.

D. Messages exchanged

Another metric to compare is the number of messages exchanged by different algorithms. Here we ignore the messages exchanged in the information exchange period where each node learns about its neighbors since it is performed for all the algorithms. Further we leave out the CTA algorithm since it is a centralized algorithm. We compare the number of messages exchanged by the MAC Scheduling of [9], conflict resolution phase of CBH, and the adaptation in the LH based algorithms. We assume one message contains all the required information to be sent, and there are no retransmissions involved. The messages used for broadcasting the schedule length (Phase-2) in MAC Scheduling are also not counted. In MAC Scheduling since each node broadcasts its schedule to two-hops, the number messages exchanged of a node is equal to 1 + degree of the node. In CBH, apart from 1-hop schedule distribution two messages are exchanged for every conflicting edge - one message for proposal by a node and a reply message by the other node with a decision of what schedule to use. It is possible that more than two messages are exchanged for the same edge since the decision could still result in a conflict. In LH based heuristics, there is no schedule distribution but only adapts the edges with invalid schedules with a valid schedule, and for each such adaptation two messages are exchanged, similar to the CBH. When the number of available channels are aplenty (in Scenarios 1 and 3), the LH based heuristics exchange the least number of messages. However, in Scenario 4 (see Fig. 3(d)), the CBH exchanges more messages than other algorithms. This is mainly because of the lower number of available channels, and hence causing more conflicts in the drawn distributed schedule. The LH based heuristics are also affected by the lower number of available channels and the increase in density (in Scenario 4) but still outperform MAC Scheduling. We note here that the MAC Scheduling [9] has a Phase-2 where the schedule length is propagated throughout the network. This requires many messages, which have not been used for comparison. Therefore, messages exchanged in our algorithms are always less than MAC Scheduling.

VI. CONCLUSIONS

In this article we put forth the problem of allocating channels in a CRAN. We proposed to use graph theoretic techniques to find an optimal and valid allocation when the channels are limited. We proposed novel distributed heuristics that uses only local common control channel. We compared our results with popular edge coloring based allocation techniques in the literature. It was shown that the proposed heuristics outperform the one in the literature. While currently available heuristics usually perform a network-wide spectrum allocation – at each snapshot – we show that a locally adapting heuristic performs as well. We have also abstracted the effect of variability wireless channel in our simulation. While MAC Scheduling performs assignment sequentially, CBH and LH perform assignment in parallel at the nodes. Hence allocation time is much higher in MAC Scheduling compared to our algorithms. The trade-off is the number of conflicts that arise. While MAC Scheduling, and CBH perform network-wide spectrum allocation per each frame, LH based algorithms have low complexity with local adaptations. Further LH based algorithms have low overhead and delay. These characteristics make LH based algorithms highly desirable for implementation. Further, the solutions proposed in this article are heuristics (hence are of polynomial complexity) that can be implemented easily.

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