CS4140
Embedded Systems Laboratory

Introduction to Control Theory
Why Control Theory?

- Embedded systems integrated with appl’n
- Multi-disciplinary training required:
  - Physics engineering
  - Electronics engineering
  - Mechanical engineering
  - Control engineering
  - …
  - And, of course,
  - Computer science & engineering
Control is Everywhere

- Automotive
- Aerospace
- Plant Control
- Climate Control
- Health Care
- Copiers, Wafer Scanners
- Model Quad Rotors ...
Cruise Control

v controller

\( e \) \( v_{\text{setp}} \)

\( T \)

\( v_{\text{meas}} \)

engine++

speedometer

vehicle

\( F_{\text{vehicle}} \)

road, air

\( v_{\text{vehicle}} \)

disturbances (slope, wind)

\( e = \) enable [0/1]

\( T = \) throttle [%]

\( F = \) thrust [N]

\( v = \) velocity [m/s]

\( v_{\text{setp}} = \) setpoint

\( v_{\text{meas}} = \) measured

\( v_{\text{vehicle}} = \) actual

CS4140 ESL (2017-2018)
Objectives of this Crash Course

- Appreciate the benefits of control
- Understand basic control principles
- Communicate with control engineers

- Get you up to speed to do the QR control
Part I: Feedback Control

- What is Control
- The Feedback Loop
- Proportional Feedback
Velocity Control

- \( e \) (1)
- \( v_{\text{setp}} \)
- \( v_{\text{meas}} \)
- control function: try to maintain \( v_{\text{meas}} = v_{\text{setp}} \)

\[
T \downarrow \quad (4) \quad \downarrow \quad (3)
\]

\[
\text{v controller} \quad \text{engine}++ \quad \text{vehicle} \quad \text{speedometer} \quad \text{road, air} \quad \text{disturbances (slope, wind)} \quad (1)
\]

\( \downarrow \quad (5) \)

\( \uparrow \quad (2) \)

\( \uparrow \quad (6) \)
Feedback Control Loop

controller function $T = h_c(\varepsilon)$: adjust $T$ such that $\varepsilon \to 0$

control theory: how to determine function $h_c$
Standard Loop Format

standard form: control $h_s$ through $h_c$ such that $y = x$

$h_c = h_{controller}$
$h_s = h_{system}$
Proportional Control

Let $h_c(\varepsilon) = P \varepsilon$

(Steady-state) Analysis:

Let $h_s(a) = ca$ (i.e. linear system)
Then $y = cP(x-y) \Rightarrow y = \left(\frac{cP}{cP+1}\right)x$
Effect of Loop Gain

\[ y = \frac{P}{P+1} x \]

Loop gain: the larger, the better (\(y \approx x\))
Example: Velocity Control

Analysis:

\[ v_{\text{meas}} = h_{\text{speedometer}}(v_{\text{vehicle}}) \]

If \( P \gg 1 \) then

\[ v_{\text{meas}} \approx v_{\text{setp}} \]

Consequently,

\[ v_{\text{vehicle}} \approx h_{\text{speedometer}}^{-1}(v_{\text{setp}}) \]

Ideally, \( h_{\text{speedometer}}(x) = x \)

Result:

\[ v_{\text{vehicle}} \approx v_{\text{setp}} \]
Example: Variable Amplifier

Analysis:

If $PA \gg 1$ (i.e. sufficient loop gain) then $z \approx x$

Hence $y \approx \frac{1}{A} x$ (e.g. $A = 0.001 \Rightarrow 1000 \times$ amp)
Part II: Blessings of Feedback

- High Loop Gain: More Robustness
- High Loop Gain: More Linearity
- High Loop Gain: More Speed
More Robustness

Suppose \( h_s \) varies with time

\[
0.98 
\quad + 
\quad 0.02 
\quad \rightarrow 
\quad 49 
\quad \rightarrow 
\quad 1 
\quad \rightarrow 
\quad 49/50 = 0.98
\]

\[
0.978 
\quad + 
\quad 0.022 
\quad \rightarrow 
\quad 49 
\quad \rightarrow 
\quad 0.9 
\quad \rightarrow 
\quad 44.1/45.1 = 0.978 !
\]

10% change in \( h_s \): only \( 10%/50 = 0.2\% \) change in \( y \)
Example: Velocity Control

For sufficiently high loop gain: \( v_{\text{meas}} \) stable \((\approx v_{\text{setp}})\),

Hence \( v_{\text{vehicle}} \approx h_{\text{speedometer}}^{-1}(v_{\text{setp}}) \), which is stable
More Linearity

Suppose $h_s$ is non-linear function

Analysis:

Let $h_s(a) = c_a a \Rightarrow y = \frac{(c_a P)/(c_a P+1)}{x}$

If $c_a P \gg 1$ then $y \approx x \Rightarrow y$ is linear with $x$
Example: Audio Amp

Analysis:

If $c_a \cdot P \cdot A \gg 1$ then $v \approx v_{in}$
Hence $v_{out} \approx 1/A \cdot v_{in}$ (so linear gain: $1/A$)
More Speed

Vehicle response (slow):
\[10 \frac{dv(t)}{dt} + v(t) = T(t)\]

Let \(T(t) = 1\) =>
\[v(t) = 1 - e^{-t/10}\]

T and v are typically time-varying signals (function of t). Transfer function (h) is not just a proportional gain function but a first-order transfer function:
Example: Velocity Control

In 2 steps of 100 ms same level (0.74) as 10 s w/o feedback
Performance of vehicle has effectively increased ~50 times!
Part III: Harnessing Feedback

- Instability Problem
- Classical Control Theory
Loop Gain Limitations

Analysis: \( y = \frac{P}{P+1} \times \)

Problem:
P should be infinite for control error to become zero
In practice however, loop gain must be limited for stability
Example 1: Integrator Systems

\[ dp(t)/dt = K(t) \quad d\phi(t)/dt = p(t) \]

\[ P = 1 \]

\[ \int K \, p \, \phi \]

\[ \phi_s \]

\[ \sin \omega t \]

\[ \sin \omega t \]

\[ -\cos \omega t \]

\[ -\sin \omega t \]

\[ 0 \]

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Example 2: Time Latency

Let $h_s: y(t) = a(t-0.5)$ (i.e., 0.5s delay)

Phase lag of 180 deg at 1 Hz causes instability
Phase Lag: examples

- **Integration (90 deg):**
  - speed -> position, flow -> volume

- **First-order system (up to 90 deg):**
  - lamp, heating, car velocity, ...

- **N-th order system (up to N*90 deg):**
  - compositions of 1st-order systems, missiles

- **Delay systems (unlimited):**
  - humans, computers, sample times, cables, air

Need **control theory** to analyze, e.g., control stability
Describe $x(t)$, $y(t)$, $h_c(t)$, $h_s(t)$ in terms of their Laplace transforms $X(s)$, $Y(s)$, $H_c(s)$, $H_s(s)$, respectively.

\[ L[f(t)] = F(s) = \int_{0}^{\infty} f(t)e^{-st} \, dt \]
For linear system \( h \) it holds \( Y(s) = H(s) \cdot X(s) \) (i.e. composition in time domain reduces to multiplication in the Laplace domain). This allows for easy analysis.
Laplace cheat sheet

- \( L[a] = \frac{a}{s} \)
- \( L[\text{at}] = \frac{a}{s^2} \)
- \( L[a f + b g] = a L[f] + b L[g] \)
- \( L[f'] = s F(s) - f(0) \)
Example: Rate Control (1)

Let \( x(t) = 1 \)
\[ \Rightarrow \]
\[ X(s) = \frac{1}{s} \]
\[ Y(s) = H(s) X(s) = \frac{1}{s^2} \]
\[ \Rightarrow \]
\[ y(t) = t \]

\[ \frac{dy(t)}{dt} = x(t) \]

Laplace transform:
\[ s \, Y(s) = X(s) \]
\[ H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s} \]

Let \( x(t) = 1 \)
\[ \Rightarrow \]
\[ X(s) = \frac{1}{s} \]
\[ Y(s) = H(s) \, X(s) = \frac{1}{s^2} \]
\[ \Rightarrow \]
\[ y(t) = t \]
Control System Analysis

\[ Y(s) = H(s) \cdot X(s) \]

**Stability:**
- \( \text{Re}(\text{roots } H(s)) < 0 \)
- \( \text{Im}(\text{roots } H(s)) \) small
Example: Rate Control (2)

\[ Y(s) = P \frac{H(s)}{1 + P H(s)} X(s) = H_1(s) X(s) \]

\[ H(s) = \frac{1}{s} \]

\[ H_1(s) = \frac{P}{s + P} \]

First-order system with time constant \(1/P\)

(root: \(s = -P \Rightarrow Re < 0, Im = 0\) so stable)
Part IV: QR Control

- Instability Problem
- Cascaded P Control
Rate control using P controller

P controller for roll rate:

\[
\frac{dp(t)}{dt} = K(t)
\]

\[\int \]

P < 1: useless control performance

P ≥ 1: stable (for not too high P!)
Angle control using P controller

P controller for roll angle:

\[ \phi_s + \frac{dp(t)}{dt} = K(t) \]

\[ \frac{d\phi(t)}{dt} = p(t) \]

\[ P < 1: \text{useless control performance} \]

\[ P \geq 1: \text{instability} \]
Angle control using cascaded P control

Embedded rate controller “neutralizes” one integrator

Cascaded P Controller: stable (for not too high P1 and P2!)

[kalman_control.pdf]
Summary

- Feedback control offers many advantages
- Is ubiquitous (cars, planes, missiles, QRs ..)
- Potential stability problems
- Need control theory
- This was merely introduction into the field
- Get a feel by applying to QR!